

1. Koje se od matrica mogu pomnožiti? Izračunati moguće proizvode.

$$\text{a) } A = \begin{bmatrix} -4 & 3 & 0 \\ 2 & -5 & 3 \end{bmatrix}, B = [-2 \quad 5], C = \begin{bmatrix} -3 & 5 \\ 7 & -4 \\ 0 & 1 \\ -2 & 0 \end{bmatrix}, D = \begin{bmatrix} -4 & 2 & 0 & 3 \\ 1 & -7 & 6 & -2 \\ 2 & 0 & -3 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} -4 & 3 & 0 \\ 2 & -5 & 3 \end{bmatrix}, B = [-2 \quad 5], C = \begin{bmatrix} -3 & 5 \\ 7 & -4 \\ 0 & 1 \\ -2 & 0 \end{bmatrix}, D = \begin{bmatrix} -4 & 2 & 0 & 3 \\ 1 & -7 & 6 & -2 \\ 2 & 0 & -3 & -1 \end{bmatrix}$$

$$AD = \begin{bmatrix} -4 & 3 & 0 \\ 2 & -5 & 3 \end{bmatrix} \begin{bmatrix} -4 & 2 & 0 & 3 \\ 1 & -7 & 6 & -2 \\ 2 & 0 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 19 & -29 & 18 & -18 \\ -7 & 39 & -39 & 13 \end{bmatrix}$$

$$BA = [-2 \quad 5] \begin{bmatrix} -4 & 3 & 0 \\ 2 & -5 & 3 \end{bmatrix} = [18 \quad -31 \quad 15]$$

$$CA = \begin{bmatrix} -3 & 5 \\ 7 & -4 \\ 0 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} -4 & 3 & 0 \\ 2 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 22 & -34 & 15 \\ -36 & 41 & -12 \\ 2 & -5 & 3 \\ 8 & -6 & 0 \end{bmatrix}$$

$$DC = \begin{bmatrix} -4 & 2 & 0 & 3 \\ 1 & -7 & 6 & -2 \\ 2 & 0 & -3 & -1 \end{bmatrix} \begin{bmatrix} -3 & 5 \\ 7 & -4 \\ 0 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 20 & -28 \\ -48 & 39 \\ -4 & 7 \end{bmatrix}$$

$$\text{b) } A = \begin{bmatrix} -3 \\ 4 \\ 2 \\ -5 \end{bmatrix}, B = [-1 \quad 3], C = \begin{bmatrix} -3 & 5 \\ 7 & -6 \end{bmatrix}, D = \begin{bmatrix} 0 & -2 & 5 & -4 \\ 1 & -5 & 3 & 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 \\ 4 \\ 2 \\ -5 \end{bmatrix} [-2 \quad 5] = \begin{bmatrix} 6 & -15 \\ -8 & 20 \\ -4 & 10 \\ 10 & -25 \end{bmatrix}$$

$$BC = [-2 \quad 5] \begin{bmatrix} -3 & 5 \\ 7 & -6 \end{bmatrix} = [41 \quad -40]$$

$$BD = [-2 \quad 5] \begin{bmatrix} 0 & -2 & 5 & -4 \\ 1 & -5 & 3 & 6 \end{bmatrix} = [5 \quad -21 \quad 5 \quad 38]$$

$$CC = \begin{bmatrix} -3 & 5 \\ 7 & -6 \end{bmatrix} \begin{bmatrix} -3 & 5 \\ 7 & -6 \end{bmatrix} = \begin{bmatrix} 44 & -45 \\ -63 & 71 \end{bmatrix}$$

$$CD = \begin{bmatrix} -3 & 5 \\ 7 & -6 \end{bmatrix} \begin{bmatrix} 0 & -2 & 5 & -4 \\ 1 & -5 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -19 & 0 & 42 \\ -6 & 16 & 17 & -64 \end{bmatrix}$$

$$DA = \begin{bmatrix} 0 & -2 & 5 & -4 \\ 1 & -5 & 3 & 6 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 22 \\ -47 \end{bmatrix}$$

c) $A = \begin{bmatrix} -3 \\ 0 \\ -2 \\ 5 \end{bmatrix}, B = \begin{bmatrix} 0 & -5 & 3 & 3 \\ -7 & 6 & 0 & 1 \end{bmatrix}, C = [-1 \ 5 \ -2], D = \begin{bmatrix} -4 & 3 \\ 2 & -3 \\ 0 & -1 \end{bmatrix}$

$$AC = \begin{bmatrix} -3 \\ 0 \\ -2 \\ 5 \end{bmatrix} \begin{bmatrix} -1 & 5 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -15 & 6 \\ 0 & 0 & 0 \\ 2 & -10 & 4 \\ -5 & 25 & -10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -5 & 3 & 3 \\ -7 & 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 26 \end{bmatrix}$$

$$CD = [-1 \ 5 \ -2] \begin{bmatrix} -4 & 3 \\ 2 & -3 \\ 0 & -1 \end{bmatrix} = [14 \ -16]$$

$$DB = \begin{bmatrix} -4 & 3 \\ 2 & -3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -5 & 3 & 3 \\ -7 & 6 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -21 & 38 & -12 & -9 \\ 21 & -28 & 6 & 3 \\ 7 & -6 & 0 & -1 \end{bmatrix}$$

d) $A = \begin{bmatrix} -4 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -7 \end{bmatrix}, B = [-1 \ -7 \ 2], C = \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix}, D = \begin{bmatrix} 8 & 0 & -7 \\ 0 & -4 & 3 \end{bmatrix}$.

$$AA = \begin{bmatrix} -4 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -7 \end{bmatrix} \begin{bmatrix} -4 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 14 & 14 & -4 \\ -7 & 17 & -20 \\ 5 & -50 & 59 \end{bmatrix}$$

$$AC = \begin{bmatrix} -4 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -7 \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -22 \\ 13 \\ -19 \end{bmatrix}$$

$$BA = [-1 \quad -7 \quad 2] \begin{bmatrix} -4 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -7 \end{bmatrix} = [-3 \quad 33 \quad -28]$$

$$BC = [-1 \quad -7 \quad 2] \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix} = [5]$$

$$CB = \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix} [-1 \quad -7 \quad 2] = \begin{bmatrix} -6 & -42 & 12 \\ 1 & 7 & -2 \\ -2 & -14 & 4 \end{bmatrix}$$

$$DA = \begin{bmatrix} 8 & 0 & -7 \\ 0 & -4 & 3 \end{bmatrix} \begin{bmatrix} -4 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -7 \end{bmatrix} = \begin{bmatrix} -32 & -51 & 49 \\ -4 & 27 & -29 \end{bmatrix}$$

$$DC = \begin{bmatrix} 8 & 0 & -7 \\ 0 & -4 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 34 \\ 10 \end{bmatrix}$$

2. Za matricu $A = \begin{bmatrix} 2 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$ izračunaj A^2 i A^3 .

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 \\ -1 & 17 & -8 \\ 5 & -20 & 11 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 2 & 2 & -4 \\ -1 & 17 & -8 \\ 5 & -20 & 11 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 0 \\ 1 & -3 & 2 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & -2 & -30 \\ 15 & -89 & 42 \\ -10 & 5 & -51 \end{bmatrix}$$

3. Izračunati determinantu matrice:

$$A = \begin{bmatrix} -2 & -5 \\ 8 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -7 \\ 6 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 0 & 3 \\ -1 & 2 & 5 \\ 4 & 0 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & -6 & 0 \\ 0 & 1 & 8 \\ 7 & -3 & -3 \end{bmatrix}.$$

$$\det A = \begin{vmatrix} -2 & -5 \\ 8 & 6 \end{vmatrix} = -2 \cdot 6 - 8 \cdot (-5) = -12 + 40 = 28$$

$$\det B = \begin{vmatrix} 3 & -7 \\ 6 & -1 \end{vmatrix} = 3 \cdot (-1) - 6 \cdot (-7) = -3 + 42 = 39$$

$$\begin{aligned} \det C &= \begin{vmatrix} -2 & 0 & 3 \\ -1 & 2 & 5 \\ 4 & 0 & -2 \end{vmatrix} \begin{vmatrix} -2 & 0 \\ -1 & 2 \\ 4 & 0 \end{vmatrix} = \\ &= (-2) \cdot 2 \cdot (-2) + 0 \cdot 5 \cdot 4 + 3 \cdot (-1) \cdot 0 - (4 \cdot 2 \cdot 3 + 0 \cdot 5 \cdot (-2) + (-2) \cdot (-1) \cdot 0) = \\ &= 8 + 0 + 0 - (24 + 0 + 0) = -16 \end{aligned}$$

$$\begin{aligned} \det D &= \begin{vmatrix} 1 & -6 & 0 \\ 0 & 1 & 8 \\ 7 & -3 & -3 \end{vmatrix} \begin{vmatrix} 1 & -6 \\ 0 & 1 \\ 7 & -3 \end{vmatrix} = \\ &= 1 \cdot 1 \cdot (-3) + (-6) \cdot 8 \cdot 7 + 0 \cdot 0 \cdot (-3) - (7 \cdot 1 \cdot 0 + (-3) \cdot 8 \cdot 1 + (-3) \cdot 0 \cdot (-6)) = \\ &= -3 - 336 + 0 - (0 - 24 + 0) = -339 + 24 = -315 \end{aligned}$$

4. Izračunaj inverznu matricu, date matrice: a) $A = \begin{bmatrix} -3 & 4 \\ -2 & 2 \end{bmatrix}$, b) $C = \begin{bmatrix} 7 & 0 & -2 \\ 0 & -1 & 1 \\ 3 & 5 & -5 \end{bmatrix}$.

$$\text{a)} \quad A^{-1} = \frac{1}{\det A} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$$

$$\det A = \begin{vmatrix} -3 & 4 \\ -2 & 2 \end{vmatrix} = (-3) \cdot 2 - (-2) \cdot 4 = -6 + 8 = 2$$

$$A_{11} = (-1)^{1+1}[2] = 2 \quad A_{12} = (-1)^{1+2}[-2] = (-1) \cdot (-2) = 2$$

$$A_{21} = (-1)^{2+1}[4] = (-1) \cdot 4 = -4 \quad A_{22} = (-1)^{2+2}[-3] = -3$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ 2 & -3 \end{bmatrix}$$

$$\text{b)} \quad C^{-1} = \frac{1}{\det C} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$\det C = \begin{vmatrix} 7 & 0 & -2 \\ 0 & -1 & 1 \\ 3 & 5 & -5 \end{vmatrix} \begin{vmatrix} 7 & 0 \\ 0 & -1 \\ 3 & 5 \end{vmatrix} =$$

$$= 7 \cdot (-1) \cdot (-5) + 0 \cdot 1 \cdot 3 + (-2) \cdot 0 \cdot 5 - (3 \cdot (-1) \cdot (-2) + 5 \cdot 1 \cdot 7 + (-5) \cdot 0 \cdot 0) =$$

$$= 35 - 41 = -6$$

$$C_{11} = \begin{vmatrix} -1 & 1 \\ 5 & -5 \end{vmatrix} = 0 \quad C_{12} = - \begin{vmatrix} 0 & 1 \\ 3 & -5 \end{vmatrix} = 3 \quad C_{13} = \begin{vmatrix} 0 & -1 \\ 3 & 5 \end{vmatrix} = 3$$

$$C_{21} = - \begin{vmatrix} 0 & -2 \\ 5 & -5 \end{vmatrix} = -10 \quad C_{22} = \begin{vmatrix} 7 & -2 \\ 3 & -5 \end{vmatrix} = -29 \quad C_{23} = - \begin{vmatrix} 7 & 0 \\ 3 & 5 \end{vmatrix} = -35$$

$$C_{31} = \begin{vmatrix} 0 & -2 \\ -1 & 1 \end{vmatrix} = -2 \quad C_{32} = - \begin{vmatrix} 7 & -2 \\ 0 & 1 \end{vmatrix} = -7 \quad C_{33} = \begin{vmatrix} 7 & 0 \\ 0 & -1 \end{vmatrix} = -7$$

$$C^{-1} = \frac{1}{-6} \begin{vmatrix} 0 & -10 & -2 \\ 3 & -29 & -7 \\ 3 & -35 & -7 \end{vmatrix}$$

5. Rešiti sistem jednačina:

- Metodom zamene

$$\begin{array}{l} x + 2y - z = 2 \\ 1) \quad 3x - y + 2z = 7 \\ \quad 4x + y + z = 5 \end{array} \quad \begin{array}{l} x + 2y - 6z = -13 \\ 2) \quad 2x + 5y + 4z = 24 \\ \quad 3x + 10y + z = 26 \end{array}$$

1)

$$\begin{array}{l} x + 2y - z = 2 \Rightarrow x = -2y + z + 2 \\ 3x - y + 2z = 7 \\ \hline 4x + y + z = 5 \end{array}$$

$$\begin{array}{l} x + 2y - z = 2 \\ 3(-2y + z + 2) - y + 2z = 7 \\ \hline 4(-2y + z + 2) + y + z = 5 \end{array}$$

$$\begin{array}{l} x + 2y - z = 2 \\ -6y + 3z + 6 - y + 2z = 7 \\ \hline -8y + 4z + 8 + y + z = 5 \end{array}$$

$$\begin{array}{l} x + 2y - z = 2 \\ -7y + 5z = 1 \\ \hline -7y + 5z = -3 \end{array}$$

Kod druge i treće jednačine, leva strana je ista, a desna se razlikuje, pa sistem nije saglasan, odnosno nema rešenja.

2)

$$\begin{array}{l} x + 2y - 6z = -13 \Rightarrow x = -2y + 6z - 13 \\ 2x + 5y + 4z = 24 \\ \hline 3x + 10y + z = 26 \end{array}$$

$$\begin{array}{l} x + 2y - 6z = -13 \Rightarrow x = -2y + 6z - 13 \\ 2x + 5y + 4z = 24 \\ \hline 3x + 10y + z = 26 \end{array}$$

$$\begin{aligned}
 x + 2y - 6z &= -13 \\
 2(-2y + 6z - 13) + 5y + 4z &= 24 \\
 3(-2y + 6z - 13) + 10y + z &= 26
 \end{aligned}$$

$$\begin{aligned}
 x + 2y - 6z &= -13 \\
 y + 16z &= 50 \Rightarrow y = -16z + 50 \\
 4y + 19z &= 65
 \end{aligned}$$

$$\begin{aligned}
 x + 2y - 6z &= -13 \\
 y + 16z &= 50 \\
 4(-16z + 50) + 19z &= 65
 \end{aligned}$$

$$\begin{aligned}
 x + 2y - 6z &= -13 \\
 y + 16z &= 50 \\
 -45z = -135 \Rightarrow z &= \frac{135}{45} = 3
 \end{aligned}$$

$$\begin{aligned}
 x + 2y - 6z &= -13 \\
 y + 16 \cdot 3 &= 50 \Rightarrow y = 50 - 48 = 2 \\
 z &= 3
 \end{aligned}$$

$$\begin{aligned}
 x + 2 \cdot 2 - 6 \cdot 3 &= -13 \Leftrightarrow x = -4 + 18 - 13 = 1 \\
 y &= 2 \\
 z &= 3
 \end{aligned}$$

Sistem je saglasan. Ima jedinstveno rešenje (1, 2, 3)

- **Gausovom metodom**

$$\begin{array}{ll}
 3x + 2y + z = 5 & x + y + z = 3 \\
 3) \quad 2x + 3y - 5z = 8 & 4) \quad x - 2y + 2z = 1 \\
 \quad 5x + y - 8z = 7 & \quad -2x + y - 3z = 4
 \end{array}$$

3)

$$\begin{aligned}
 3x + 2y + z &= 5 \quad /I \cdot 5 + II, / \quad I \cdot 8 + III \\
 2x + 3y - 5z &= 8 \\
 5x + y - 8z &= 7
 \end{aligned}$$

Prvu jednačinu množimo sa 5 i dodajemo drugoj jednačini, zatim prvu jednačinu množimo sa 8 i dodajemo trećoj jednačini. Tako iz druge i treće jednačine eliminišemo promenljivu z .

$$\begin{aligned}
 3x + 2y + z &= 5 \\
 17x + 13y - 5z + 5z &= 25 + 8 \\
 29x + 17y - 8z + 8z &= 40 + 7
 \end{aligned}$$

$$\begin{array}{l}
 3x + 2y + z = 5 \\
 17x + 13y = 33 \quad /II \cdot (-17) + III \cdot 13 \\
 29x + 17y = 47
 \end{array}$$

Drugu jednačinu množimo sa -17 , a treću sa 13 , pa ih saberemo. Iz treće jednačine smo eliminisali i promenljivu y .

$$\begin{array}{l}
 3x + 2y + z = 5 \\
 17x + 13y = 33 \\
 -17 \cdot 17x + -17 \cdot 13y + 13 \cdot 29x + 13 \cdot 17y = -17 \cdot 33 + 13 \cdot 47
 \end{array}$$

$$\begin{array}{l}
 3x + 2y + z = 5 \\
 17x + 13y = 33 \\
 -289x - 221y + 377x + 221y = -561 + 611
 \end{array}$$

$$\begin{array}{l}
 3x + 2y + z = 5 \\
 17x + 13y = 33 \\
 88x = 50 \Rightarrow x = \frac{50}{88} = \frac{25}{44}
 \end{array}$$

$$\begin{array}{l}
 3x + 2y + z = 5 \\
 17 \cdot \frac{25}{44} + 13y = 33 \Rightarrow y = \frac{79}{44} \\
 x = \frac{25}{44}
 \end{array}$$

$$\begin{array}{l}
 3 \cdot \frac{25}{44} + 2 \cdot \frac{79}{44} + z = 5 \Rightarrow z = -\frac{13}{44} \\
 y = \frac{79}{44} \\
 x = \frac{25}{44}
 \end{array}$$

Sistem je saglasan. Ima jedinstveno rešenje $(x, y, z) = \left(\frac{25}{44}, \frac{79}{44}, -\frac{13}{44}\right)$.

- Kramerovom metodom

$$\begin{array}{ll}
 3x + 2y + z = 5 & 2x + y + z = -1 \\
 5) \quad 2x + 3y + z = 1 & 6) \quad -x + y + 2z = 3 \\
 2x + y + 3z = 11 & 3x + 2y + 4z = 1
 \end{array}$$

5)

$$\begin{array}{l}
 3x + 2y + z = 5 \\
 2x + 3y + z = 1 \\
 2x + y + 3z = 11
 \end{array}$$

$$D = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 27 + 4 + 2 - 6 - 3 - 12 = 12$$

$$D_x = \begin{vmatrix} 5 & 2 & 1 \\ 1 & 3 & 1 \\ 11 & 1 & 3 \end{vmatrix} \begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix} = 45 + 22 + 1 - 33 - 5 - 6 = 24$$

$$D_y = \begin{vmatrix} 3 & 5 & 1 \\ 2 & 1 & 1 \\ 2 & 11 & 3 \end{vmatrix} \begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} = 9 + 10 + 22 - 2 - 33 - 30 = -24$$

$$D_z = \begin{vmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \\ 2 & 1 & 11 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 99 + 4 + 10 - 30 - 3 - 44 = 36$$

$$x = \frac{D_x}{D} = \frac{24}{12} = 2 \quad y = \frac{D_y}{D} = \frac{-24}{12} = -2 \quad z = \frac{D_z}{D} = \frac{36}{12} = 3$$

Sistem je saglasan. Ima jedinstveno rešenje $(x, y, z) = (2, -2, 3)$.

6)

$$2x + y + z = -1$$

$$-x + y + 2z = 3$$

$$3x + 2y + 4z = 1$$

$$D = \begin{vmatrix} 2 & 1 & 1 \\ -1 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ -1 & 1 \\ 3 & 2 \end{vmatrix} = 8 + 6 - 2 - 3 - 8 + 4 = 5$$

$$D_x = \begin{vmatrix} -1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 4 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ 3 & 1 \\ 1 & 2 \end{vmatrix} = -4 + 2 + 6 - 1 + 4 - 12 = -5$$

$$D_y = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 3 & 1 & 4 \end{vmatrix} \begin{vmatrix} 2 & -1 \\ -1 & 3 \\ 3 & 1 \end{vmatrix} = 24 - 6 - 1 - 9 - 4 - 4 = 0$$

$$D_z = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ -1 & 1 \\ 3 & 2 \end{vmatrix} = 2 + 9 + 2 + 3 - 12 + 1 = 5$$

$$x = \frac{D_x}{D} = \frac{-5}{5} = -1 \quad y = \frac{D_y}{D} = \frac{0}{5} = 0 \quad z = \frac{D_z}{D} = \frac{5}{5} = 1$$

Sistem je saglasan. Ima jedinstveno rešenje $(x, y, z) = (-1, 0, 1)$.

- **Matričnom metodom**

$$7) \begin{array}{l} x + 2y - z = -3 \\ 2x + 3y + z = -1 \\ x - y - z = 3 \end{array} \quad 8) \begin{array}{l} 2x - y + 3z = -1 \\ x + 2y - 4z = 5 \\ 3x + y + 2z = 1 \end{array}$$

7)

$$\begin{aligned} x + 2y - z &= -3 \\ 2x + 3y + z &= -1 \\ x - y - z &= 3 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

$$A \cdot X = B$$

Rešavamo matričnu jednačinu

$$X = A^{-1} \cdot B$$

Najpre računamo A^{-1} .

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -3 + 2 + 2 + 3 + 1 + 4 = 9$$

$$A_{11} = \begin{vmatrix} 3 & 1 \\ -1 & -1 \end{vmatrix} = -2 \quad A_{12} = - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = 3 \quad A_{13} = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -5$$

$$A_{21} = - \begin{vmatrix} 2 & -1 \\ -1 & -1 \end{vmatrix} = 3 \quad A_{22} = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = 0 \quad A_{23} = - \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = 3$$

$$A_{31} = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 5 \quad A_{32} = - \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -3 \quad A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$$

$$A^{-1} = \frac{1}{9} \begin{bmatrix} -2 & 3 & 5 \\ 3 & 0 & -3 \\ -5 & 3 & -1 \end{bmatrix}$$

$$X = A^{-1} \cdot B = \frac{1}{9} \begin{bmatrix} -2 & 3 & 5 \\ 3 & 0 & -3 \\ -5 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 18 \\ -18 \\ 9 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Sistem je saglasan. Ima jedinstveno rešenje $(x, y, z) = (2, -2, 1)$.

8)

$$2x - y + 3z = -1$$

$$x + 2y - 4z = 5$$

$$3x + y + 2z = 1$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & -4 \\ 3 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}$$

$$A \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -4 \\ 3 & 1 & 2 \end{vmatrix} \begin{vmatrix} 2 & -1 \\ 1 & 2 \\ 3 & 1 \end{vmatrix} = 8 + 12 + 3 - 18 + 8 + 2 = 15$$

$$A_{11} = \begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix} = 8 \quad A_{12} = -\begin{vmatrix} 1 & -4 \\ 3 & 2 \end{vmatrix} = 14 \quad A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$$A_{21} = -\begin{vmatrix} -1 & 3 \\ 1 & 2 \end{vmatrix} = 5 \quad A_{22} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5 \quad A_{23} = -\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = -5$$

$$A_{31} = \begin{vmatrix} -1 & 3 \\ 2 & -4 \end{vmatrix} = -2 \quad A_{32} = -\begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = 11 \quad A_{33} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5$$

$$A^{-1} = \frac{1}{15} \begin{bmatrix} 8 & 5 & -2 \\ -14 & -5 & 11 \\ -5 & -5 & 5 \end{bmatrix}$$

$$X = A^{-1} \cdot B = \frac{1}{15} \begin{bmatrix} 8 & 5 & -2 \\ -14 & -5 & 11 \\ -5 & -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 15 \\ 0 \\ -15 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Sistem je saglasan. Ima jedinstveno rešenje $(x, y, z) = (1, 0, -1)$.

Rešiti sistem jednačina:

1) Metodom zamene

a) $\begin{array}{l} x + 3y - 5z = 6 \\ 2x + z = 3 \\ 3x + 3y - 4z = 9 \end{array}$ b) $\begin{array}{l} 3x + 3y - 5z = 6 \\ 2x + 2y + z = 3 \\ 3x + 3y - 4z = 9 \end{array}$ c) $\begin{array}{l} x + y = 6 \\ 2x + y = 9 \\ 4x + 2y = 18 \end{array}$ d) $\begin{array}{l} x + y + z = 5 \\ 3x - y + 2z = 1 \end{array}$

2) Gausovom metodom

a) $\begin{array}{l} x - y + 3z = 20 \\ -3x + 4y + 2z = -7 \\ -x + 2y + z = -2 \end{array}$ rešenje $(3, -2, 5)$ b) $\begin{array}{l} x - 3y + 4z = 14 \\ -x + 2y - 5z = -13 \\ 2x + 5y - 3z = -5 \end{array}$ rešenje $(4, -2, 1)$

c) $\begin{array}{l} x + 2y - 5z = 6 \\ -2x + y + 2z = 5 \\ -3x + 3y - 4z = 8 \end{array}$ rešenje $(1, 5, 1)$ d) $\begin{array}{l} 2x - 3y + z = -9 \\ 5x + y - 2z = 12 \\ x - 2y - 3z = 1 \end{array}$ rešenje $(1, 3, -2)$

3) Kramerovom metodom

a) $\begin{array}{l} x + 3y - 2z = 1 \\ 2x - y + z = 3 \\ x + 2z = 7 \end{array}$ rešenje $(1, 2, 3)$ b) $\begin{array}{l} x + 2y - 3z = 0 \\ 2x + 4y - 6z = 1 \\ x + y + z = 4 \end{array}$ c) $\begin{array}{l} x + 2y + 3z = 6 \\ 4x + 5y + 6z = 15 \\ 7x + 8y + 8z = 23 \end{array}$

4) pomoću matričnih jednačina

a) $\begin{array}{l} 2x - 3y + z = -1 \\ x + y + z = 6 \\ 3x + y - 2z = -1 \end{array}$ b) $\begin{array}{l} 2x - 3y + z = 2 \\ 3x - 5y + 5z = 3 \\ 5x - 8y + 6z = 5 \end{array}$ c) $\begin{array}{l} 3x - y + 3z = 4 \\ 6x - 2y + 6z = 1 \\ 5x + 4y = 2 \end{array}$